

Table 10.12 INTEGRALS OF AIRY FUNCTIONS

| x | $\int_0^x \text{Ai}(t) dt$ | $\int_0^x \text{Ai}^*(-t) dt$ | $\int_0^x \text{Bi}(t) dt$ | $\int_0^x \text{Bi}^*(-t) dt$ | x | $\int_0^x \text{Ai}(t) dt$ | $\int_0^x \text{Ai}^*(-t) dt$ | $\int_0^x \text{Bi}^*(-t) dt$ |
|-----|---|---|---|---|------|---|---|---|
| 0.0 | 0.00000 00 | 0.00000 00 | 0.00000 00 | 0.00000 00 | 5.0 | 0.33328 76 | 0.71788 22 | 0.15873 09 |
| 0.1 | 0.03421 01 | 0.03679 54 | 0.06373 67 | 0.05924 87 | 5.1 | 0.33329 73 | 0.75103 62 | 0.14113 39 |
| 0.2 | 0.06585 15 | 0.07615 70 | 0.13199 45 | 0.11398 10 | 5.2 | 0.33330 50 | 0.77926 27 | 0.11667 30 |
| 0.3 | 0.09497 09 | 0.11802 51 | 0.20487 68 | 0.16411 57 | 5.3 | 0.33331 11 | 0.80111 58 | 0.08660 41 |
| 0.4 | 0.12164 06 | 0.16229 44 | 0.28256 70 | 0.20952 89 | 5.4 | 0.33331 59 | 0.81545 49 | 0.05250 03 |
| 0.5 | 0.14595 33 | 0.20880 95 | 0.36533 85 | 0.25006 28 | 5.5 | 0.33331 97 | 0.82151 82 | +0.01617 86 |
| 0.6 | 0.16801 77 | 0.25736 07 | 0.45356 50 | 0.28553 62 | 5.6 | 0.33332 27 | 0.81897 90 | -0.02038 99 |
| 0.7 | 0.18795 52 | 0.30768 05 | 0.54773 36 | 0.31575 56 | 5.7 | 0.33332 50 | 0.80797 96 | -0.05518 54 |
| 0.8 | 0.20589 45 | 0.35944 15 | 0.64845 82 | 0.34052 58 | 5.8 | 0.33332 69 | 0.78914 06 | -0.08625 18 |
| 0.9 | 0.22196 97 | 0.41225 56 | 0.75649 64 | 0.35966 27 | 5.9 | 0.33332 83 | 0.76354 19 | -0.11181 25 |
| 1.0 | 0.23631 73 | 0.46567 40 | 0.87276 91 | 0.37300 50 | 6.0 | 0.33332 95 | 0.73267 53 | -0.13038 11 |
| 1.1 | 0.24907 33 | 0.51918 94 | 0.99838 41 | 0.38042 77 | 6.1 | 0.33333 03 | 0.69836 93 | -0.14086 00 |
| 1.2 | 0.26037 12 | 0.57224 05 | 1.13466 38 | 0.38185 43 | 6.2 | 0.33333 10 | 0.66268 96 | -0.14262 05 |
| 1.3 | 0.27034 09 | 0.62421 79 | 1.28318 00 | 0.37726 99 | 6.3 | 0.33333 16 | 0.62781 93 | -0.13555 73 |
| 1.4 | 0.27910 66 | 0.67447 31 | 1.44579 42 | 0.36673 34 | 6.4 | 0.33333 20 | 0.59592 62 | -0.12011 15 |
| 1.5 | 0.28678 67 | 0.72232 88 | 1.62470 81 | 0.35038 81 | 6.5 | 0.33333 23 | 0.56902 35 | -0.09726 08 |
| 1.6 | 0.29349 24 | 0.76709 26 | 1.82252 33 | 0.32847 24 | 6.6 | 0.33333 25 | 0.54883 59 | -0.06847 29 |
| 1.7 | 0.29932 75 | 0.80807 24 | 2.04231 52 | 0.30132 67 | 6.7 | 0.33333 27 | 0.53667 65 | -0.03562 42 |
| 1.8 | 0.30438 82 | 0.84459 41 | 2.28772 12 | 0.26939 97 | 6.8 | 0.33333 29 | 0.53334 74 | -0.00088 80 |
| 1.9 | 0.30876 29 | 0.87602 06 | 2.56304 90 | 0.23325 04 | 6.9 | 0.33333 30 | 0.53906 98 | +0.03340 40 |
| 2.0 | 0.31253 28 | 0.90177 28 | 2.87340 83 | 0.19354 74 | 7.0 | 0.33333 31 | 0.55345 17 | 0.06491 67 |
| 2.1 | 0.31577 11 | 0.92135 09 | 3.21007 37 | 0.15106 46 | 7.1 | 0.33333 31 | 0.57549 72 | 0.09147 36 |
| 2.2 | 0.31854 43 | 0.93435 56 | 3.56000 00 | 0.10667 18 | 7.2 | 0.33333 32 | 0.60365 96 | 0.11121 47 |
| 2.3 | 0.32091 19 | 0.94050 97 | 3.91400 00 | 0.06132 23 | 7.3 | 0.33333 32 | 0.63593 60 | 0.12273 90 |
| 2.4 | 0.32292 74 | 0.93967 67 | 4.27300 00 | +0.01603 45 | 7.4 | 0.33333 33 | 0.66999 96 | 0.12521 80 |
| 2.5 | 0.32463 80 | 0.93187 78 | 4.63600 00 | -0.02812 94 | 7.5 | 0.33333 33 | 0.70336 19 | 0.11847 31 |
| 2.6 | 0.32608 57 | 0.91730 54 | 5.00000 00 | -0.07009 01 | 7.6 | 0.33333 33 | 0.73355 34 | 0.10300 57 |
| 2.7 | 0.32730 74 | 0.89633 20 | 5.36400 00 | -0.10878 06 | 7.7 | 0.33333 33 | 0.75830 99 | 0.07997 85 |
| 2.8 | 0.32833 55 | 0.86951 37 | 5.72800 00 | -0.14317 88 | 7.8 | 0.33333 33 | 0.77755 13 | 0.05114 35 |
| 2.9 | 0.32919 83 | 0.83758 77 | 6.09200 00 | -0.17234 20 | 7.9 | 0.33333 33 | 0.78453 65 | +0.01872 22 |
| 3.0 | 0.32992 04 | 0.80146 29 | 6.45600 00 | -0.19544 25 | 8.0 | 0.33333 33 | 0.78398 26 | -0.01475 64 |
| 3.1 | 0.33052 31 | 0.76220 32 | 6.82000 00 | -0.21180 21 | 8.1 | 0.33333 33 | 0.77413 57 | -0.04664 84 |
| 3.2 | 0.33102 49 | 0.72100 37 | 7.18400 00 | -0.22092 49 | 8.2 | 0.33333 33 | 0.75578 55 | -0.07440 43 |
| 3.3 | 0.33144 15 | 0.67915 91 | 7.54800 00 | -0.22252 61 | 8.3 | 0.33333 33 | 0.73041 93 | -0.09577 87 |
| 3.4 | 0.33178 65 | 0.63802 56 | 7.91200 00 | -0.21655 57 | 8.4 | 0.33333 33 | 0.70011 70 | -0.10902 22 |
| 3.5 | 0.33207 15 | 0.59897 71 | 8.27600 00 | -0.20321 50 | 8.5 | 0.33333 33 | 0.66739 21 | -0.11303 86 |
| 3.6 | 0.33230 63 | 0.56335 61 | 8.64000 00 | -0.18296 47 | 8.6 | 0.33333 33 | 0.63499 08 | -0.10749 35 |
| 3.7 | 0.33249 93 | 0.53242 25 | 9.00400 00 | -0.15652 33 | 8.7 | 0.33333 33 | 0.60566 32 | -0.09285 98 |
| 3.8 | 0.33265 76 | 0.50730 05 | 9.36800 00 | -0.12485 43 | 8.8 | 0.33333 33 | 0.58192 70 | -0.07039 64 |
| 3.9 | 0.33278 70 | 0.48892 77 | 9.73200 00 | -0.08914 28 | 8.9 | 0.33333 33 | 0.56584 22 | -0.04205 63 |
| 4.0 | 0.33289 27 | 0.47800 75 | 10.09600 00 | -0.05076 01 | 9.0 | 0.33333 33 | 0.55881 97 | -0.01033 04 |
| 4.1 | 0.33297 86 | 0.47496 79 | 10.46000 00 | -0.01121 78 | 9.1 | 0.33333 33 | 0.56148 12 | +0.02196 26 |
| 4.2 | 0.33304 84 | 0.47992 95 | 10.82400 00 | +0.02788 79 | 9.2 | 0.33333 33 | 0.57358 51 | 0.05192 24 |
| 4.3 | 0.33310 50 | 0.49268 51 | 11.18800 00 | 0.06494 00 | 9.3 | 0.33333 33 | 0.59403 00 | 0.07682 93 |
| 4.4 | 0.33315 07 | 0.51269 28 | 11.55200 00 | 0.09837 02 | 9.4 | 0.33333 33 | 0.62093 76 | 0.09439 87 |
| 4.5 | 0.33318 76 | 0.53908 35 | 11.91600 00 | 0.12673 04 | 9.5 | 0.33333 33 | 0.65181 01 | 0.10300 27 |
| 4.6 | 0.33321 73 | 0.57068 59 | 12.28000 00 | 0.14876 50 | 9.6 | 0.33333 33 | 0.68375 25 | 0.10183 70 |
| 4.7 | 0.33324 11 | 0.60606 63 | 12.64400 00 | 0.16347 66 | 9.7 | 0.33333 33 | 0.71373 85 | 0.09101 44 |
| 4.8 | 0.33326 02 | 0.64358 51 | 13.00800 00 | 0.17018 59 | 9.8 | 0.33333 33 | 0.73889 84 | 0.07157 33 |
| 4.9 | 0.33327 54 | 0.68146 70 | 13.37200 00 | 0.16857 74 | 9.9 | 0.33333 33 | 0.75680 07 | 0.04539 57 |
| 5.0 | 0.33328 76 | 0.71788 22 | 13.73600 00 | 0.15873 09 | 10.0 | 0.33333 33 | 0.76569 84 | 0.01504 04 |
| | $\left[\begin{smallmatrix} (-4)3 \\ 5 \end{smallmatrix} \right]$ | $\left[\begin{smallmatrix} (-3)1 \\ 7 \end{smallmatrix} \right]$ | $\left[\begin{smallmatrix} (-3)4 \\ 7 \end{smallmatrix} \right]$ | $\left[\begin{smallmatrix} (-3)1 \\ 6 \end{smallmatrix} \right]$ | | $\left[\begin{smallmatrix} (-7)3 \\ 3 \end{smallmatrix} \right]$ | $\left[\begin{smallmatrix} (-3)1 \\ 7 \end{smallmatrix} \right]$ | $\left[\begin{smallmatrix} (-3)1 \\ 7 \end{smallmatrix} \right]$ |

Table 10.13 ZEROS AND ASSOCIATED VALUES OF AIRY FUNCTIONS AND THEIR DERIVATIVES

| s | a_s | $\text{Ai}'(a_s)$ | a'_s | $\text{Ai}(a'_s)$ | b_s | $\text{Bi}'(b_s)$ | b'_s | $\text{Bi}(b'_s)$ |
|-----|---------------|-------------------|---------------|-------------------|---------------|-------------------|---------------|-------------------|
| 1 | -2.33810 741 | +0.70121 082 | -1.01879 297 | +0.53565 666 | -1.17371 322 | +0.60195 789 | -2.29443 968 | -0.45494 438 |
| 2 | -4.08794 944 | -0.80311 137 | -3.24819 758 | -0.41901 548 | -3.27109 330 | -0.76031 014 | -4.07315 509 | -0.39652 284 |
| 3 | -5.52055 983 | +0.86520 403 | -4.82009 921 | +0.38040 647 | -4.83073 784 | +0.83699 101 | -5.51239 573 | -0.36796 916 |
| 4 | -6.78670 809 | -0.91085 074 | -6.16330 736 | -0.35790 794 | -6.16985 213 | -0.88947 990 | -6.78129 445 | +0.34949 912 |
| 5 | -7.94413 359 | +0.94733 571 | -7.37217 726 | +0.34230 124 | -7.37676 208 | +0.92998 364 | -7.94017 869 | -0.33602 624 |
| 6 | -9.02265 085 | -0.97792 281 | -8.48848 673 | -0.33047 623 | -8.49194 885 | -0.96323 443 | -9.01958 336 | +0.32550 974 |
| 7 | -10.04017 434 | +1.00437 012 | -9.53544 905 | +0.32102 229 | -9.53819 438 | +0.99158 637 | -10.03769 633 | -0.31693 465 |
| 8 | -11.00852 430 | -1.02773 869 | -10.52766 040 | -0.31318 539 | -10.52991 351 | -1.01638 966 | -11.00646 267 | +0.30972 594 |
| 9 | -11.93601 556 | +1.04872 065 | -11.47505 663 | +0.30651 729 | -11.47695 355 | +1.03849 429 | -11.93426 165 | -0.30352 766 |
| 10 | -12.82877 675 | -1.06779 386 | -12.38478 837 | -0.30073 083 | -12.38641 714 | -1.05847 184 | -12.82725 831 | +0.29810 491 |

AUXILIARY TABLE—COMPLEX ZEROS AND ASSOCIATED VALUES OF $\text{Bi}(z)$ AND $\text{Bi}'(z)$

| s | $e^{-i/s}$ | $\text{Bi}'(e_s)$ | $e^{-i/s}$ | $\text{Bi}(e'_s)$ |
|-----|---------------|-------------------|---------------|-------------------|
| | Modulus Phase | Modulus Phase | Modulus Phase | Modulus Phase |
| 1 | 2.354 0.095 | 0.993 +2.641 | 1.121 0.331 | 0.750 +0.466 |
| 2 | 4.093 0.042 | 1.136 -0.513 | 3.257 0.059 | 0.592 -2.632 |
| 3 | 5.524 0.027 | 1.224 +2.625 | 4.824 0.033 | 0.538 +0.515 |
| 4 | 6.789 0.020 | 1.288 -0.519 | 6.166 0.023 | 0.506 -2.624 |
| 5 | 7.946 0.015 | 1.340 +2.622 | 7.374 0.017 | 0.484 +0.519 |

From J. C. P. Miller, The Airy integral, British Assoc. Adv. Sci. Mathematical Tables Part-vol. B. Cambridge Univ. Press, Cambridge, England, 1946 and F. W. J. Olver, The asymptotic expansion of Bessel functions of large order. Philos. Trans. Roy. Soc. London [A] 247, 328-368, 1954 (with permission).

*See page 11.